Appendix B

α — The Thermalization Parameter

In addition to the NLTE lines, the models include, self consistently, line blanketing of the metal lines selected from the latest atomic and ionic line list of Kurucz. The source function for these lines can be written as follows (Mihalas, 1978):

$$S_l = \frac{\int \Phi_\nu J_\nu d\nu + \epsilon' B_\nu(T_e) + \eta B^*(T_r)}{1 + \epsilon' + \eta} \tag{B.1}$$

The first term of the numerator is represents the scattering reservoir, the second the thermal source (photons created by collisional excitation followed by radiative de-excitation) and the third term are those photons created by recombination from the continuum to the upper level followed by a radiative de-excitation. In the denominator, ϵ represents those photons destroyed by collisional de-excitation and η is a sink term proportional to the total rate of ionization from the upper state times the fractional recombination rate to the lower level. $B_\nu(T_e)$ is the Planck function based on the local electron temperature, T_e, and $B^*(T_r)$ resembles the Planck function for a characteristic radiation temperature, T_r.

The following is a derivation of ϵ based on the work of Castor (1970).

$$\epsilon = \frac{\epsilon'}{1+\epsilon'} \tag{B.2}$$

$$\epsilon' = \frac{C_{ul}(1 - exp(-u))}{A_{ul}} \tag{B.3}$$

Where $u = \frac{h\nu}{kT}$ and $E_o = h\nu$.

with $C_{ul} = (\frac{n_l}{n_u})^* C_{lu}$ and $\frac{g_l}{g_u} = (\frac{n_l}{n_u})^* exp(-u)$

Given

$$C_{lu} = C_o n_e T^{\frac{1}{2}} \left[14.5 f_{lu} \left(\frac{I_H}{E_o} \right)^2 \right] uexp(-u)\Gamma(u) \tag{B.4}$$

and

$$A_{ul} = \frac{64\pi^4\nu^3}{3hc^3 g_u} \frac{g_l f_{lu} 3he^2}{8\pi^2 m_e \nu} \tag{B.5}$$

with

$$C_o = \pi a_o^2 (\frac{8k}{m_e \pi})^{\frac{1}{2}} \tag{B.6}$$

then ϵ' can be rewritten as:

$$\epsilon' = \frac{C_o[14.5(\frac{I_H}{E_o})^2] u\Gamma(u)(1 - exp(-u)) m_e c^3 n_e T^{\frac{1}{2}}}{8\pi^2\nu^2 e^2} \tag{B.7}$$

$$\epsilon' = \frac{a_o^2 [14.5 I_H^2] m_e^{\frac{1}{2}} c^3}{\sqrt{8}\pi^{\frac{3}{2}} e^2 h k^{\frac{1}{2}}} \frac{n_e}{T^{\frac{1}{2}}\nu^3} \Gamma(u)(1 - exp(-u)) \tag{B.8}$$

$$\epsilon' = 20 \frac{\lambda^3 n_e}{T^{\frac{1}{2}}} \Gamma(u)(1 - exp(-u)) \tag{B.9}$$

$$\epsilon' = 20\frac{\lambda_\mu^3 n_{e_{10}}}{T_5^{\frac{1}{2}}}\Gamma(u)(1 - exp(-u)) \tag{B.10}$$

$\Gamma(u)$ is defined as follows $\Gamma(u) = max[\bar{g}, 0.276exp(u)E_1(u)]$, with $\bar{g} = 0.2$ for $n'l' \to nl$ and $\bar{g} = 0.7$ for $nl' \to nl$. In Fig. B.1 we can see a plot of ϵ as a function of optical depth and velocity. Note that ϵ is low over much of the atmosphere except at depth.

The the third term in the numerator, η, is defined as follows:

$$\eta = \frac{(R_{u\kappa} + C_{u\kappa})n_l^*(R_{\kappa l} + C_{l\kappa}) - g_l(R_{l\kappa} + C_{l\kappa})n_u^*(R_{\kappa u} + C_{u\kappa})/g_u}{A_{ul}[n_l^*(R_{\kappa l} + C_{l\kappa}) + n_u^*(R_{\kappa u} + C_{u\kappa})]} \tag{B.11}$$

with

$$B^* = \left(\frac{2h\nu^3}{c^2}\right)\left\{\left(\frac{n_l^* g_u}{n_u^* g_l}\right)\left[\frac{(R_{u\kappa} + C_{u\kappa})(R_{\kappa l} + C_{l\kappa})}{(R_{l\kappa} + C_{l\kappa})(R_{\kappa u} + C_{u\kappa})}\right] - 1\right\}^{-1} \tag{B.12}$$

It's clear from observation of eqn. B.12 that B^* is very similar to the Planck function and can be thought of as having a characteristic radiation temperature of T_r which is set by the photoionization and recombination rates. At depth, $R_{l\kappa} \to R_{\kappa l}$ and $R_{u\kappa} \to R_{\kappa u}$ so that $B^* \to B_\nu(T_e)$. However, in optically thin regions the results are quite different. In a SN Ia's outer atmosphere collisions are near negligible and $R_{l\kappa} > R_{\kappa l}$ and $R_{\kappa u} > R_{u\kappa}$. This is the case of a relatively *cool* medium being irradiated by a *hot* radiation field. Therefore, $B^* > B_\nu$ and S_l will be larger than the S_l determined by only the coupling to the thermal pool. Thus η increases the overall 'thermalization' in the atmosphere. For simplicity and computational expediency, we now rewrite eqn. B.1 as:

$$S_l \approx (1 - \alpha)\int \Phi_\nu J_\nu d\nu + \alpha B_\nu(T) \tag{B.13}$$

$$\text{assuming} \quad B^* \approx B_\nu(T) \tag{B.14}$$

where α now represents both the effects of ϵ and η. Since the calculation of α would require a full NLTE treatment of all lines and continua, which is beyond our present ability to acquire atomic data, we adopt an average value for α for all the metal lines in LTE. Due to the velocity gradients in SNe Ia the shape of the lines does not depend heavily on α, however, α does have an effect on the overall color of the spectrum. Therefore, it is necessary to treat α as a parameter to fit in concert with the temperature.

In Fig. B.2 and Fig. B.3 we examine the effect of α on the synthetic spectra for models which are purely in LTE. Of importance to us is how much the color and absolute magnitude are effected by varying α, since increasing α causes more thermalization and thus reddens the spectrum. In Fig. B.2 we see a very good fit to the spectrum of SN 1992A at 5 days after maximum light using an $\alpha = 0.1$. Fig. B.3 shows two unacceptable fits to the spectrum with $\alpha = 0.3$ and $\alpha = 0.01$. In Table B.1 we display the results of this parameterized study. Of particular note is the fact that the range of acceptable fits spans only $\pm$ 0.05 in color and $\pm$ 0.17 in M_V. These can be thought of as limits to our accuracy in determining the characteristics of a particular model due to the effects of α.

Model	α	T_{eff}	$B-V$	$M_V - M_V(361)$	*Grade*
355	0.01	8000	-0.08	0.65	D
353	0.05	8500	0.14	0.19	B
361	0.10	8900	0.19	0.00	A
321	0.20	9250	0.25	-0.14	B
341	0.30	9400	0.30	-0.21	D

Table B.1: The color and magnitude differences for a set of models with different values of α.

Figure B.4 compares the spectra for $\alpha = (10^{-4}, 0.05, 0.1, 1.0)$ for the W7 model considered in Chapter 4. This is a model which has almost all of the major species treated in NLTE. The UV (where most lines are treated in NLTE) is rather insensitive to the choice of α; however, in the optical, redward of 5000 Å there is a strong dependence on α. Based on these results and the LTE comparisons we choose $\alpha = 0.05 - 0.1$ as our range for SNe Ia.

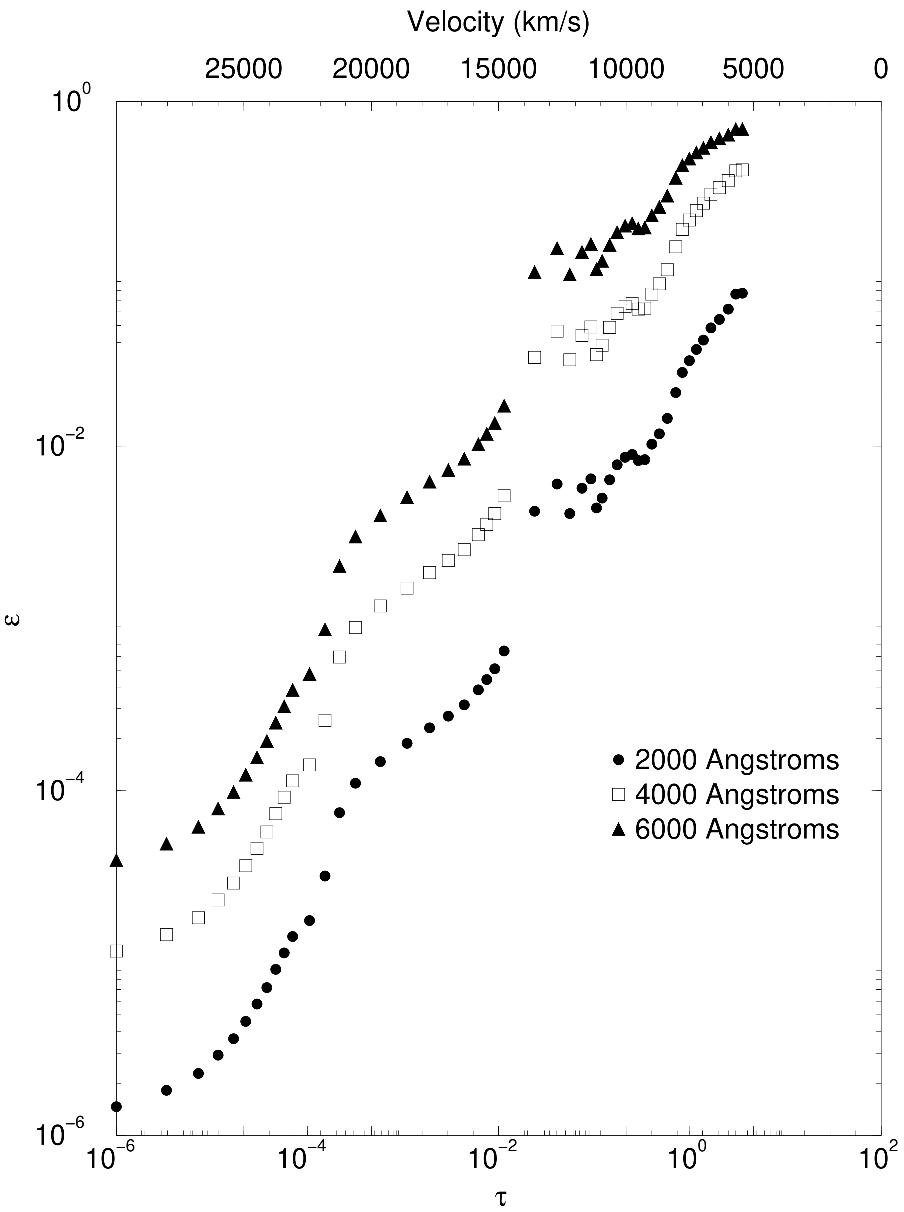

Figure B.1: ϵ as a function of wavelength for W7 at 20 days after explosion.

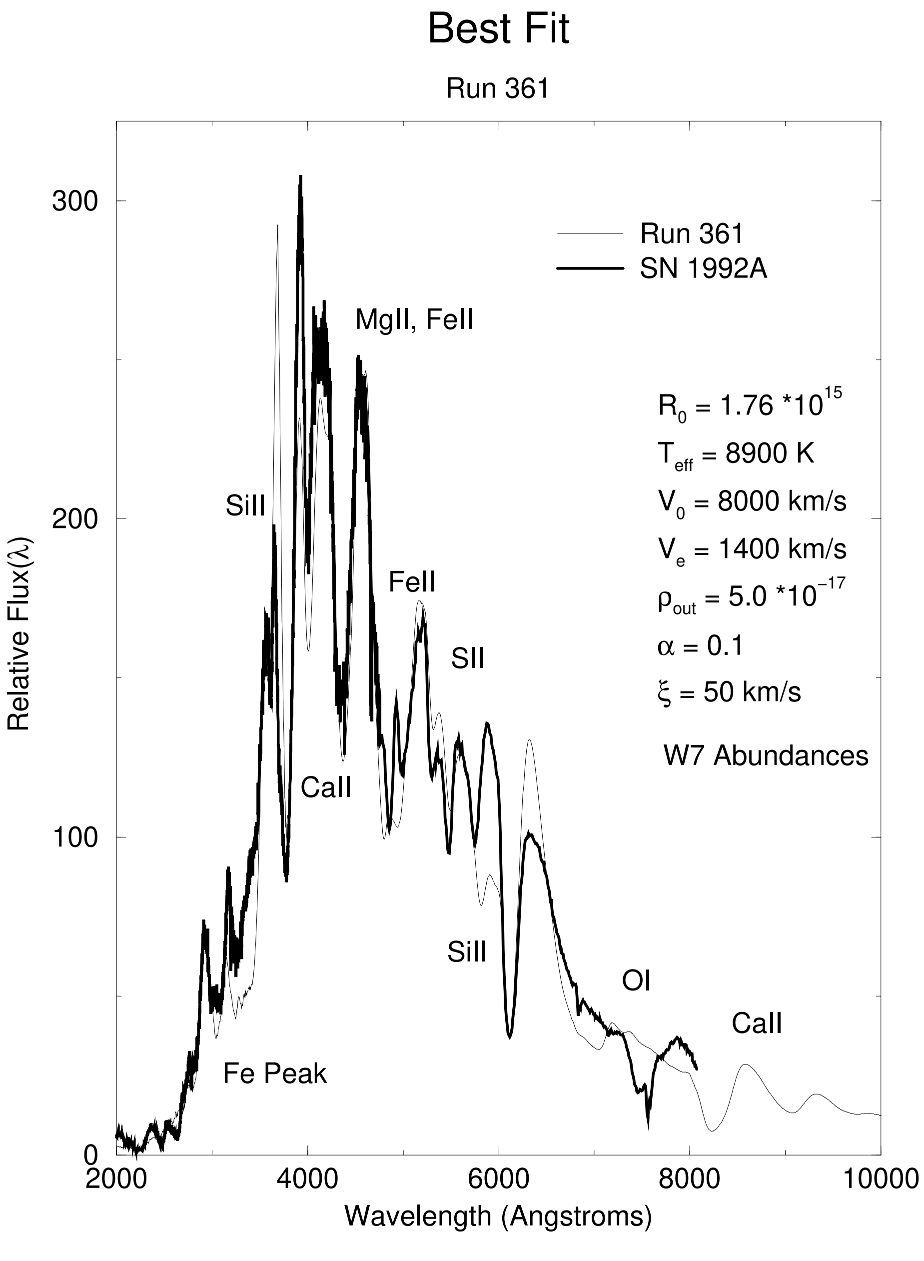

Figure B.2: A good fit to the spectrum of SN 1992A for $\alpha = 0.1$.

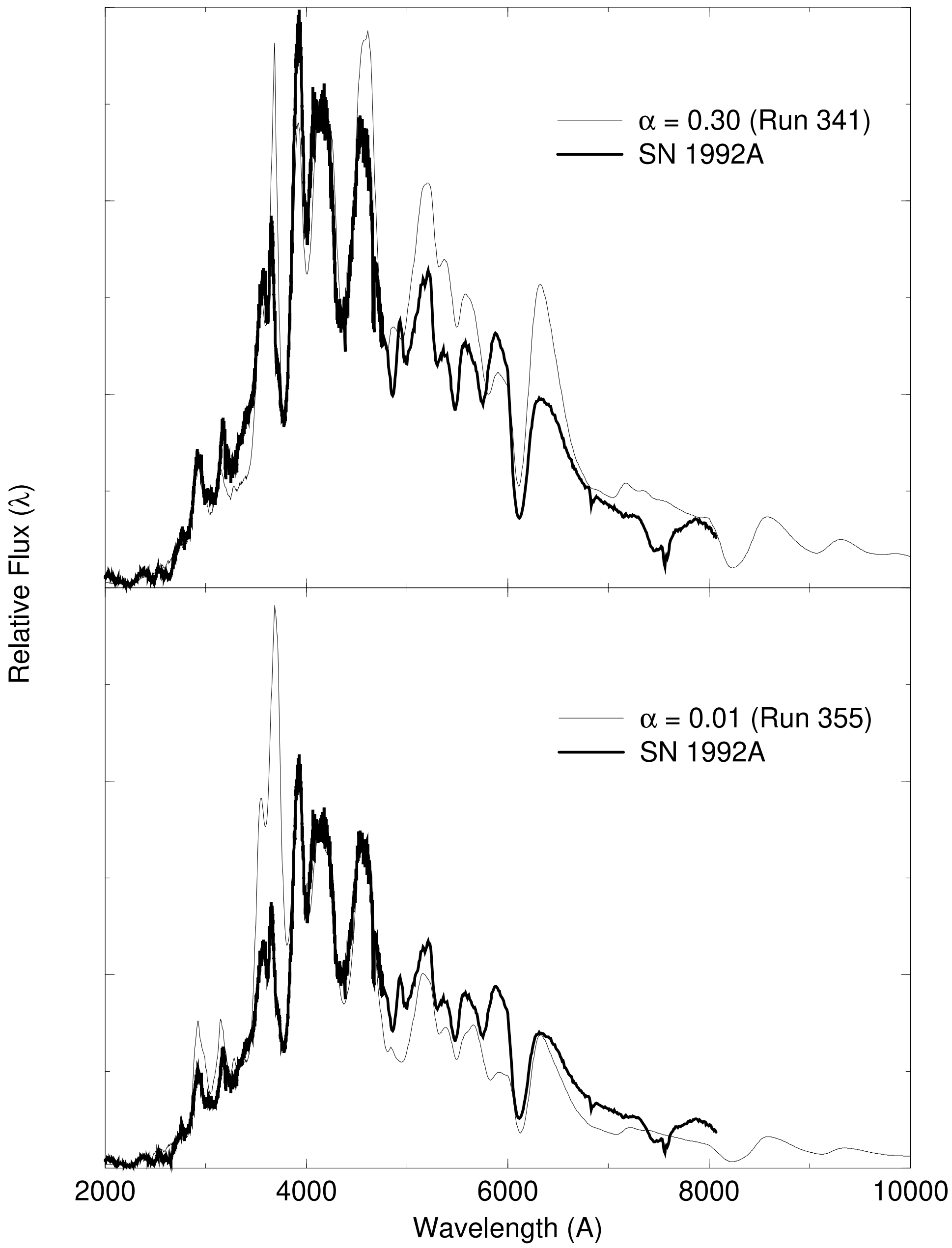

Figure B.3: Unacceptable fits to the spectrum of SN 1992A for $\alpha = 0.01$ and 0.3.

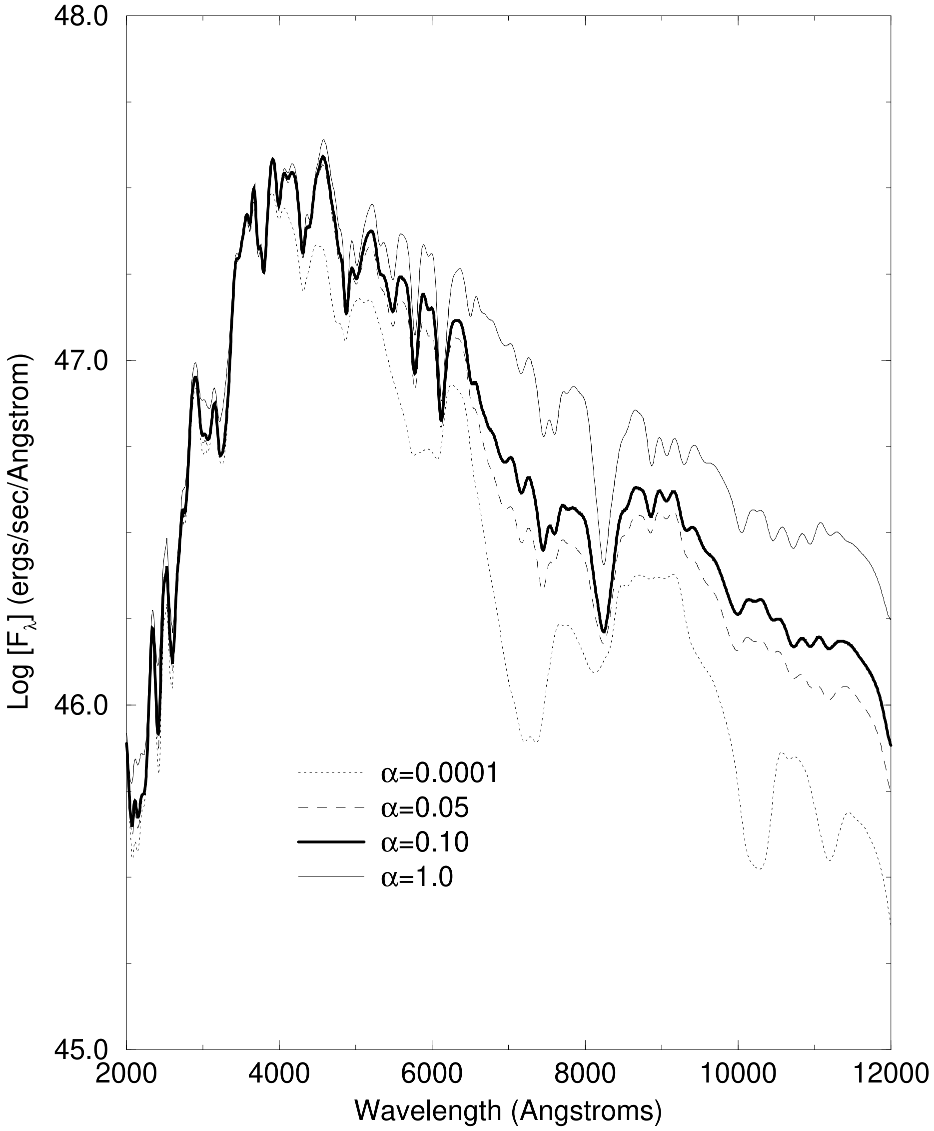

Figure B.4: The synthetic spectrum for the W7 model at 23 d past explosion for four choices of α. The model with $\alpha = 0.10$ is in radiative equilibrium and the temperature structure has been held fixed at that structure for all the models displayed.