- $\bar{F}$ is the true mean flux in the mocks, and $\xi_\delta = \langle \delta_1 \delta_2 \rangle$ the true correlation of the delta field $F = \bar{F}(1+\delta)$
- when analyzing the spectra, we compute an estimate of $\bar{F}$

$$\hat{\bar{F}} = \frac{1}{N}\sum_i^N F_i = \bar{F}\,\frac{1}{N}\sum_i (1+\delta_i) = \bar{F}\left[1+\hat{\bar{\delta}}\right] \qquad \hat{\bar{\delta}} = \frac{1}{N}\sum_i \delta_i$$

obviously $\langle \hat{\bar{\delta}} \rangle = 0$ and $\langle \hat{\bar{F}} \rangle = \bar{F}$.

- the variance that we measure is

$$\hat{\sigma}_F^2 = \frac{1}{N}\sum_i F_i^2 - \left(\hat{\bar{F}}\right)^2 = \frac{\bar{F}^2}{N}\sum_i \left(1+2\delta_i+\delta_i^2\right) - \left(\hat{\bar{F}}\right)^2 =$$

$$= \bar{F}^2\left(1+2\hat{\bar{\delta}} + \frac{1}{N}\sum_i \delta_i^2\right) - \bar{F}^2\left(1+2\hat{\bar{\delta}} + \left(\hat{\bar{\delta}}\right)^2\right) =$$

$$= \bar{F}^2\,\hat{\sigma}_\delta^2 \qquad \text{where} \qquad \hat{\sigma}_\delta^2 = \frac{1}{N}\sum_i \delta_i^2 - \left(\frac{1}{N}\sum_i \delta_i\right)^2$$

again $\langle \hat{\sigma}_\delta^2 \rangle = \sigma_\delta^2 = \langle \delta^2 \rangle$ and $\langle \hat{\sigma}_F^2 \rangle = \bar{F}^2\,\sigma_\delta^2 = \langle F^2 \rangle$

- the correlation that we measure is:

$$\hat{\xi}_F^\alpha = \frac{1}{N_\alpha}\sum_{i,j\in\alpha} F_i F_j = \bar{F}^2\left[1+2\tilde{\delta}^\alpha + \hat{\xi}_\delta^\alpha\right]$$

where N_α is the number of pixel pairs that fall into the bin α

$$\hat{\xi}_\delta^\alpha = \frac{1}{N_\alpha}\sum_{i,j\in\alpha} \delta_i \delta_j$$

BUT NOTE THAT $\boxed{\tilde{\delta}^\alpha \equiv \frac{1}{N_\alpha}\sum_{i,j\in\alpha} \delta_i \neq \hat{\bar{\delta}} \equiv \frac{1}{N}\sum_i \delta_i}$

still, we have again $\langle \hat{\xi}_\delta^\alpha \rangle = \xi_\delta(r_\alpha)$ $\quad \langle \hat{\xi}_F^\alpha \rangle = \bar{F}^2\left(1+\xi_\delta(r_\alpha)\right)$ $\quad \langle \tilde{\delta}^\alpha \rangle = 0$

. Moreover what we do is define a new variable

$$X_i \equiv \frac{F_i}{\hat{\bar{F}}} - 1 = \frac{\bar{F}}{\hat{\bar{F}}}(1+\delta_i) - 1 = (1+\varepsilon)(1+\delta_i) - 1 = \varepsilon + \delta_i + \varepsilon\delta_i$$

$$= \delta_i(1+\varepsilon) + \varepsilon$$

where $\bar{F} = \hat{\bar{F}}(1+\varepsilon)$ $\qquad \langle \varepsilon \rangle = 0$

$$\hat{\bar{X}} = \frac{\hat{\bar{F}}}{\hat{\bar{F}}} - 1 = 0 \quad \text{by construction}$$

. Its variance is

$$\hat{\sigma}_X^2 = \frac{1}{N}\sum_i X_i^2 - \left(\frac{1}{N}\sum_i X_i\right)^2 = \frac{1}{N}\sum_i \left(\delta_i(1+\varepsilon) + \varepsilon\right)^2 = \varepsilon^2 + 2(1+\varepsilon)\hat{\bar{\delta}} + (1+\varepsilon)^2\hat{\sigma}_\delta^2$$

but since $\bar{F} = \hat{\bar{F}}(1+\varepsilon) = \bar{F}(1+\hat{\bar{\delta}})(1+\varepsilon) \longrightarrow (1+\hat{\bar{\delta}})(1+\varepsilon) = 1$

$$\hat{\sigma}_X^2 = (1+\varepsilon)^2\hat{\sigma}_\delta^2 + \varepsilon^2 + 2(1+\varepsilon)(1+\hat{\bar{\delta}}) - 2(1+\varepsilon) =$$

$$= (1+\varepsilon)^2\hat{\sigma}_\delta^2 + \varepsilon^2 - 2\varepsilon$$

again we have $\langle \hat{\sigma}_X^2 \rangle = \sigma_\delta^2$

. Actually, when averaged over many mocks, the term $2\varepsilon(1+\varepsilon)\tilde{\delta}_\alpha$ goes to 0 and we end up with

$$\boxed{\hat{\xi}_X^\alpha = \varepsilon^2 + (1+\varepsilon)^2\hat{\xi}_\delta^\alpha}$$

. Its correlation is

$$\hat{\xi}_X^\alpha = \frac{1}{N_\alpha}\sum_{ij\in\alpha} X_i X_j = \frac{1}{N_\alpha}\sum_{ij\in\alpha}\left[\delta_i(1+\varepsilon)+\varepsilon\right]\left[\delta_j(1+\varepsilon)+\varepsilon\right] =$$

$$= \varepsilon^2 + 2\varepsilon(1+\varepsilon)\tilde{\delta}^\alpha + (1+\varepsilon)^2\hat{\xi}_\delta^\alpha$$

and since $\tilde{\delta}^\alpha \neq \hat{\delta}$ we can't simplify this anymore

again, though $\langle \hat{\xi}_X^\alpha \rangle = \langle \hat{\xi}_\delta^\alpha \rangle = \xi_\delta(r_\alpha)$

. If $\tilde{\delta} = \hat{\bar{\delta}}$, then we could simplify to

$$\hat{\xi}_X^\alpha = \varepsilon^2 + 2\varepsilon(1+\varepsilon)(1+\hat{\bar{\delta}}) - 2\varepsilon(1+\varepsilon) + (1+\varepsilon)^2\hat{\xi}_\delta^\alpha =$$

$$= \varepsilon^2 + 2\varepsilon - 2\varepsilon(1+\varepsilon) + (1+\varepsilon)^2\hat{\xi}_\delta^\alpha = (1+\varepsilon)^2\xi_\delta^\alpha - \varepsilon^2$$

BUT THIS IS NOT THE CASE !!! specially for thin bins α