Appendix A

γ-ray Transport in Type Ia Supernovae

In order to solve the rate equations in a consistent manner, PHOENIX must include the effects of non-thermal ionization. In Type Ia supernovae the non-thermal ionization is a result of the fast electrons produced when Ni^{56} and Co^{56} emit γ-rays as they decay radioactively and these γ-rays transfer their energy to the electrons in the atmosphere via Compton scattering.

Following in the footsteps of Sutherland & Wheeler (1984), we have developed a γ-ray transport code for SNe Ia. Their method is based on a purely absorptive, gray opacity and has been shown to be accurate to within 10% in the deposition function for a wide variety of supernovae models (up to 1200 days after explosion) when compared to more elaborate Monte Carlo methods. The reason this approach has been taken, as opposed to a Monte Carlo calculation, is for the sole purpose of computational expediency. A typical γ-ray deposition calculation for a SNe Ia requires approximately 15 minutes on an IBM RISC workstation, a comparable Monte Carlo Calculation takes over 24 hours. Since one would like to produce several model atmospheres in one day, this time constraint rules out the Monte Carlo approach.

We can break down the geometry of the problem as follows. Taking an arbitrary point in the atmosphere, of mass $dm = \rho dAdl$ (ρ is the density and $dAdl$ is the volume), the rate of deposition of energy at this point due to the source is

$$ds_{dep} = s(\mathbf{r})\kappa_\gamma \rho dl \frac{dA}{4\pi r^2} e^{-\tau(\mathbf{r})} \tag{A.1}$$

where the source is at $\mathbf{r}$ emitting γ-ray energy at the rate $s(\mathbf{r})$ ergs g^{-1} s^{-1}. κ_γ is the γ-ray absorption opacity and to a very good approximation has been found to be equal to $0.06Y_e$ (where Y_e is the total number of electrons per baryon) from the work of Colgate, Petschek, & Kriese (1980)). $\tau(\mathbf{r})$ is the γ-ray optical depth and is defined as follows:

$$\tau(\mathbf{r}) = \int_0^{\mathbf{r}} \kappa_\gamma \rho(s) ds. \tag{A.2}$$

The source $s(\mathbf{r})$ varies in time in accord with the exponential decay of Ni^{56} and Co^{56} and is defined by:

$$s = 3.90 \times 10^{10} e^{-\gamma_{Ni} t} + 6.78 \times 10^9 (e^{-\gamma_{Co}} - e^{-\gamma_{Ni}}) \tag{A.3}$$

where $\gamma_{Ni} = 1.32 \times 10^{-6}$ s^{-1} and $\gamma_{Co} = 1.02 \times 10^{-7}$ s^{-1} are the decay rates. The dimensionless deposition function is obtained by dividing dm and integrating over the radioactive source region:

$$d = \frac{\kappa_\gamma}{4\pi} \int d^3r \frac{1}{r^2} e^{-\tau(\mathbf{r})} \rho(\mathbf{r}). \tag{A.4}$$

Fig. A.1 is a rough schematic breakdown of how the atmosphere is set up for the absorption calculation. The supernova is divided into approximately 500 radial zones and 100 angular zones. The number of zones are picked so that the "optically thin" nature of this treatment (as seen eqn. A.1) is preserved. Given the abundances and time since explosion, the sources are calculated at each point and the emitted γ-rays are followed throughout the atmosphere in accordance with the above equations on equally spaced lines in μ.

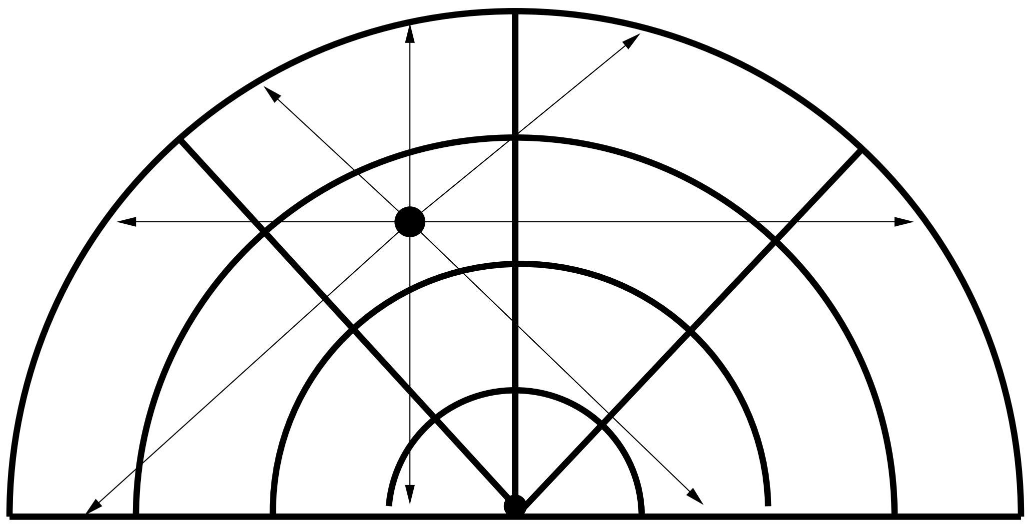

Figure A.1: Geometric lay-out for the γ-ray deposition calculation.

As a test of this method we can compare the results of the above treatment to the semi-analytic one found in Sutherland & Wheeler (1984). Here it was shown that for an atmosphere in which all the radioactive sources are located within a spherical volume $r < r_c$ then:

$$d(a) = \frac{1}{2}\int_{-1}^{\mu_{max}} d\mu[e^{-\tau_{min}} - e^{-\tau_{max}}] \tag{A.5}$$

Here, $\mu = cos(\theta)$ and tau_{min} and tau_{max} are the total optical depths to the near and far side of the radioactive core along the ray specified by $theta$ as seen in Fig. A.2. In Fig. A.3 we see how well our calculation compares to the semi-analytic result derived in Equation A.5. Note that the only discrepancy occurs in the first zone for the late time model when the analytic and numerical results differ by $\approx 5\%$.

Finally, we present the results of our deposition calculations for the three hydrodynamic explosion models for SNe Ia considered in this thesis. In Fig. A.4 we see a plot of the deposition function versus the mass fraction for models W7, WW2 and LA4 all at 20 days after explosion. Note the large differences between the C/O deflagration model W7 and the helium-igniter models WW2 and LA4.

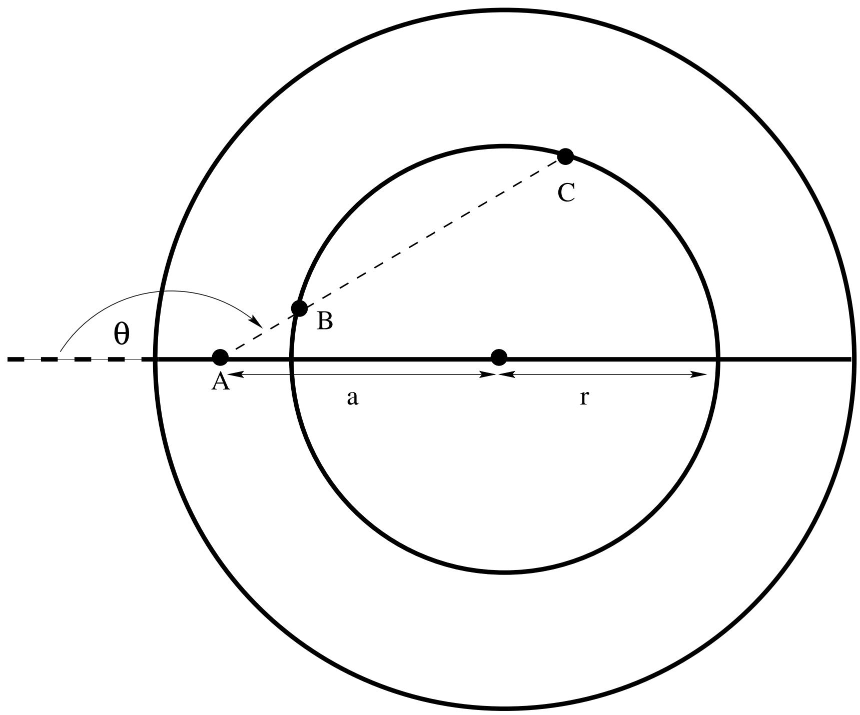

Figure A.2: The deposition is being calculated at point A (at a distance a from the center) and r is the radius of the radioactive core.

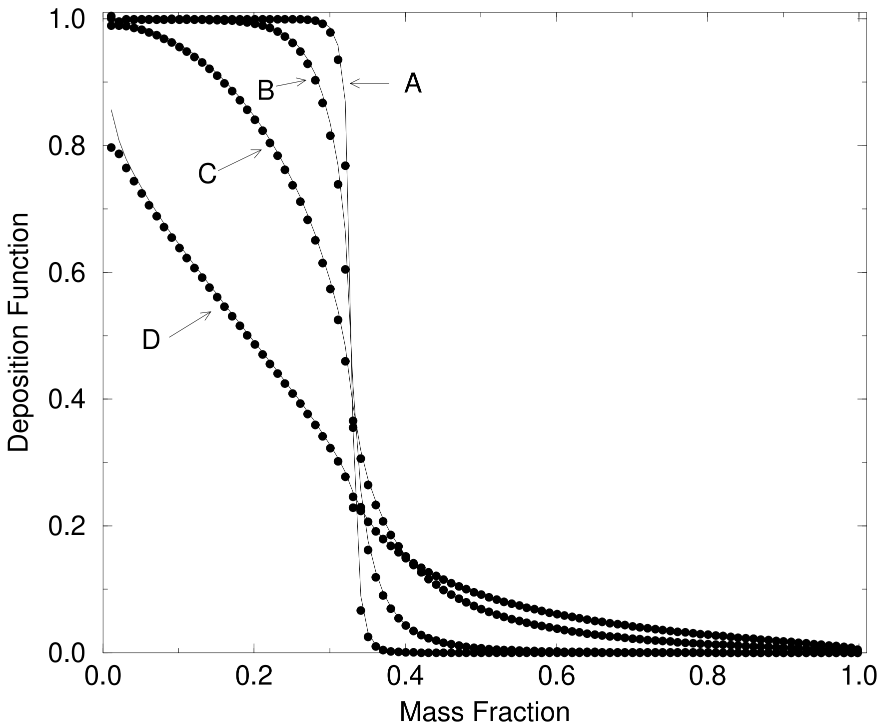

Figure A.3: A Graph of the semi-analytic deposition function (lines) vs. the numerical absorption calculation (filled circles). Curves A, B, C and D represent $t_6 = 0.5$, 1, 2 and 4 (in units of 10^6 s) respectively.

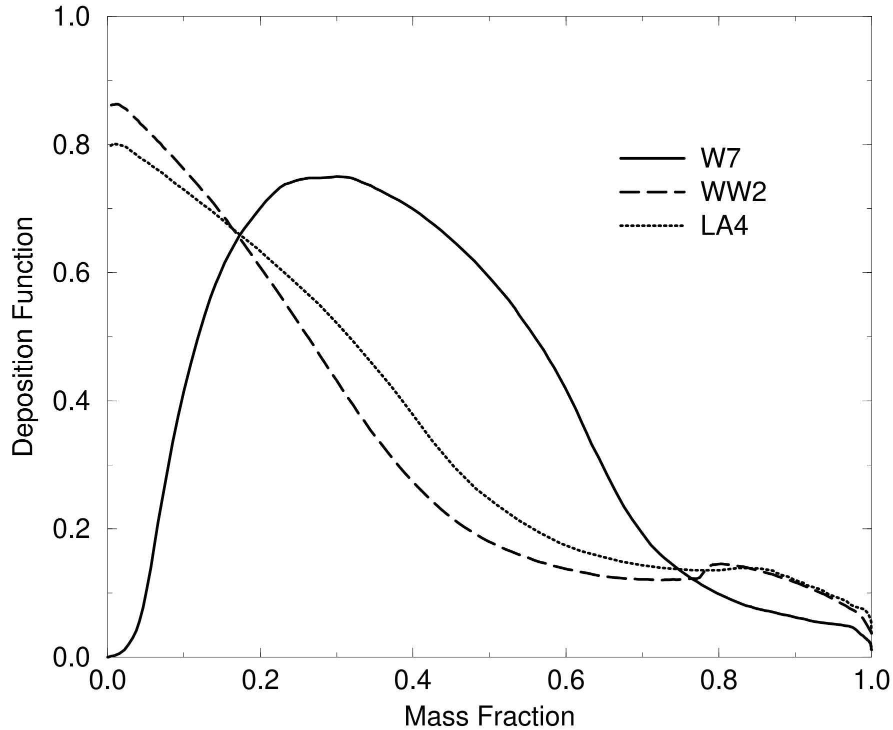

Figure A.4: The 20 day deposition functions of models W7, WW2 and LA4.