Chapter 3

Physics and the Hubble Constant

3.1 Abstract

Two physical methods for determining luminosities and distances of Type Ia supernovae, the spectral fitting expanding atmosphere method and the ^{56}Ni radioactivity method, depend on the interval between the times of explosion and maximum brightness, but in differing ways. By requiring consistency between the two methods we derive blue and visual absolute magnitudes $M_B \simeq M_V \simeq -19.47 \pm 0.45$ mag which, together with the ridge line of Type Ia supernovae in the magnitude–redshift diagram, constrain the Hubble constant to be 57 ± 11 km s^{-1} Mpc^{-1}.

3.2 Introduction

Determining the value of the Hubble constant (H_0) by means of Type Ia supernovae (SNe Ia) is currently a topic of great interest. The attractive features of SNe Ia for this purpose include (1) that nearly homogeneous subsamples of them can be isolated; (2) that they are luminous and therefore can be observed to large distances; and (3) that they are reasonably well understood physical events which can be modeled.

To compare the results of theoretical models to observations, it is necessary to account for the response of detectors. It is convenient to work in the standard astronomical convention of magnitudes. The apparent magnitude in a particular band (m_x) is defined as

$$m_x = -2.5 \log \int_0^\infty S_x(\lambda)\, F_\lambda \, d\lambda + C_x \tag{3.1}$$

where $S_x(\lambda)$ is the filter response function for band x at wavelength λ, F_λ is the observed photon flux per unit wavelength, and C_x is a normalization constant which in practice is determined by observing comparison stars whose magnitudes are known. The absolute magnitude in band x, M_x, is defined as the apparent magnitude the object would have if it were observed at a distance of 10 pc. The bolometric magnitude, $M_{\rm Bol}$, can be defined in terms of the luminosity of the sun:

$$M_{\rm Bol} = -2.5 \log \left(\frac{L}{L_\odot} \right) + M_{{\rm Bol}_\odot}, \tag{3.2}$$

where $L_\odot = 3.85 \times 10^{33}$ ergs s^{-1} is the luminosity of the sun, and $M_{{\rm Bol}_\odot} = 4.72$ mag is the absolute bolometric magnitude of the sun.

Normal SNe Ia, e.g., those which at maximum brightness have broad–band blue–minus–visual color index $|m_B - m_V| < 0.25$ mag, have intrinsic absolute magnitude dispersions $\sigma(M_B) \simeq \sigma(M_V) < 0.2$ mag Sandage & Tammann (1993); Vaughan et al. (1995, in press); Hamuy et al. (1995); Sandage & Tammann (1995, in press); Vaughan, Branch, & Perlmutter (1995, in preparation). In the magnitude–redshift diagram, normal SNe Ia form a ridge line Hamuy et al. (1995); Sandage & Tammann (1995, in press); Vaughan, Branch, & Perlmutter (1995, in preparation) which corresponds to

$$\begin{aligned} M_B &\simeq M_V \qquad (3.3) \\ &\simeq -19.75 \pm 0.10 \\ &\quad + 5 \log \left(\frac{H_0}{50 \text{ km s}^{-1} \text{ Mpc}^{-1}} \right) \text{ mag.} \end{aligned}$$

Thus, the determination of the absolute magnitude of a normal SN Ia provides the value of H_0. Furthermore, because our present understanding of SNe Ia Branch & Khokhlov (1995, in press) allows them to be modeled and their absolute magnitudes to be calculated, SNe Ia provide a determination of H_0 which is independent of all other distance determinations in astronomy.

3.3 Theory

In this chapter we combine two ways to calculate the luminosity of SNe Ia: (1) the spectral fitting expanding atmosphere method (SEAM), which is based on modeling the optical–ultraviolet spectrum, and (2) the ^{56}Ni radioactivity method, which is based on the theory that the SN Ia light curve is powered by the radioactive decay of ^{56}Ni through ^{56}Co to ^{56}Fe. These two methods depend on the rise-time t_{R}, the interval between the times of explosion and maximum brightness, in different ways. This then allows us to write down two equations involving the unknowns t_R and L and solve for them.

Given the assumptions that radioactivity powers the light curve and that near the time of maximum light most of the energy released by radioactivity is still being trapped and thermalized, the peak radiated luminosity is expected to be comparable to the instantaneous rate of energy release by radioactivity Arnett (1982); Arnett, Branch, & Wheeler (1985); Branch (1992). Thus the peak luminosity can be expressed as

$$L_{\mathrm{Ni}} = \alpha \dot{S}(t_R) M_{\mathrm{Ni}} \tag{3.4}$$

where $\dot{S}$ is the radioactivity luminosity per unit nickel mass, evaluated at time t_R, M_{Ni} is the mass of ^{56}Ni synthesized in the explosion, and α, the ratio of bolometric to radioactivity luminosities, is near unity. Using e-folding decay times for ^{56}Ni and

^{56}Co of 8.8 and 111 days, and mean energy releases per decay of 1.71 and 3.76 MeV, the radioactivity luminosity per solar mass of ^{56}Ni becomes Branch (1992):

$$\begin{aligned} \dot{S} = \quad & 6.31 \times 10^{43}\, e^{-t_{\rm R}/(8.8\,{\rm days})} \\ & +1.43 \times 10^{43}\, e^{-t_{\rm R}/(111\,{\rm days})} \quad {\rm ergs\ s^{-1}\ M_\odot^{-1}}. \end{aligned} \tag{3.5}$$

3.4 Hydrodynamical Models

In order to constrain the value of $M_{\rm Ni}$, we focus on hydrodynamical models for SNe Ia that include calculations of light curves and spectra. Höflich's Höflich (1995) work on the normal SN Ia 1994D combines detailed hydrodynamical explosion models with spectrum synthesis. This approach proves extremely useful in rejecting certain models for SNe Ia. Not only do his models have to replicate the light curve but they must also synthesize the proper elements which make up the SN Ia's atmosphere in order to properly fit the spectrum. His acceptable models for SN 1994D are M35, M36 and M37. These show a range of $0.51 \leq M_{\rm Ni} \leq 0.67\ M_\odot$. In addition to this work, Harkness Harkness (1991a); Wheeler, Harkness, & Khokhlov (1995) shows that for the normal SN Ia 1989B, model W7 of Nomoto et al. Nomoto, Thielemann, & Yokoi (1984) ($M_{\rm Ni}$=0.58 $M_\odot$) provides good fits to the series of spectra. Further he states that there are no acceptable models with $M_{\rm Ni} \leq 0.5\ M_\odot$. The light curves and spectra are sensitive to the nickel mass not only because the composition of the ejected matter directly affects the spectral lines, but also because the nickel mass determines the temperature of the ejected matter. Thus we adopt for the nickel mass the value,

$$M_{\rm Ni} = 0.6 \pm 0.2 \pm 0.1\ M_\odot, \tag{3.6}$$

where the first range covers the acceptable values found by Harkness Harkness (1991a) and Höflich Höflich (1995) and the second allows for any possible systematic effects.

For simple illustrative assumptions, including that of constant gas opacity, the value of α can be shown to be unityBranch & Khokhlov (1995, in press); Arnett (1982). Our estimate of α is based on the hydrodynamical work and light curve calculations such as presented by Khokhlov, Müller, and Höflich Khokhlov, Müller, & Höflich (1994) They find that in detailed calculations which take into account the dependence of the opacity on the temperature, α exceeds unity for models which have a sufficiently long rise time to be compatible with observation. This is due to the dependence of opacity on temperature; as the temperature falls toward 10^4 K the opacity also falls, internal energy is released rapidly, and the light curve peaks at a luminosity that exceeds the instantaneous radioactivity luminosity. For acceptable models for normal SNe Ia Branch & Khokhlov (1995, in press); Khokhlov (1995, private communication):

$$\alpha = 1.2 \pm 0.2. \tag{3.7}$$

Combining this with our $M_{\rm Ni}$ estimate we obtain:

$$\alpha M_{\rm Ni} = 0.72 \pm 0.29\ M_{\odot}. \tag{3.8}$$

The values of α and $M_{\rm Ni}$ may be anti-correlated (e.g., for a high value of $M_{\rm Ni}$, radioactivity energy released near the surface of the explosion tends to escape directly rather than to be trapped and contribute to the thermalized luminosity). By ignoring any such anti-correlation in our error estimate our range for $\alpha M_{\rm Ni}$ should be conservative.

3.5 Spectrum Synthesis

We now turn our attention to the the spectrum synthesis models of Nugent et al. Nugent et al. (1995a) (seen in Chapter 2). Accurately solving the fully relativistic radiation transport equation along with the non-LTE rate equations (for some ions) and ensuring radiative equilibrium (energy conservation) led to quality fits to the near-maximum-light spectra of SN 1992A and SN 1981B. These models allowed them to predict the supernova luminosity with the following assumptions: (1) Homologous expansion, $v \propto r$, from which it follows that the velocity of a given matter element is constant and hence that the position of a reference radius is given by $v_0 t_{\rm R}$, where v_0 is determined by the spectral fit. (2) The density gradient for the atmosphere follows a decaying exponential. (3) A high-quality fit to the spectrum implies that the model parameters were accurately determined. This method is in the spirit of the Baade method Baade (1926); Kirshner & Kwan (1974); Branch et al. (1981b) and the more detailed expanding photosphere method of Eastman and collaborators Eastman & Kirshner (1989); Schmidt, Kirshner, & Eastman (1992). To avoid confusion with photospheric based methods we refer to our present method, which entails fitting the spectrum of individual supernovae, as the Spectral-fitting Expanding Atmosphere Method (SEAM).

Clearly the luminosity depends on the rise-time through assumption (1). The fits to the normal SNe Ia 1981B and 1992A are good; thus we assume that the model parameters are determined reasonably well. From the models we find that the peak luminosity for SNe 1981B and 1992A is:

$$L_{\rm SEAM} = 1.60 \pm 0.7 \times 10^{43} \left(\frac{t_{\rm R}}{17\ {\rm days}}\right)^2\ {\rm ergs\ s^{-1}}. \tag{3.9}$$

3.6 Results

In Figure 3.1 we display the rise-time dependence of the two methods to determine the peak luminosity of normal SNe Ia. From this figure we find that L is in the range $1.0 - 2.3 \times 10^{43}$ ergs s^{-1}, which corresponds to $M_{\rm Bol} = -19.25 \pm 0.45$ mag. In order

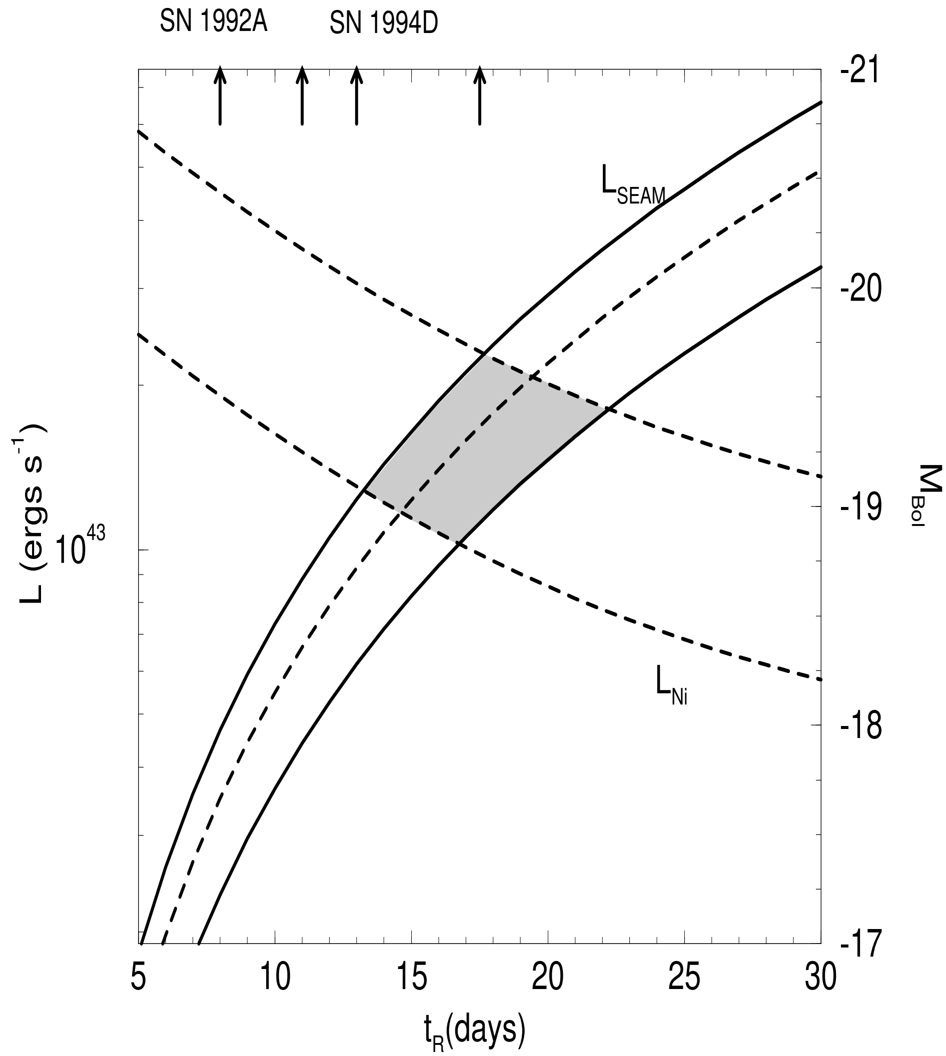

Figure 3.1: The luminosity of Type Ia supernovae as a function of rise-time. The band labeled $L_{\rm Ni}$ is the range of luminosity allowed by nickel radioactivity and the band labeled $L_{\rm SEAM}$ is the range of luminosity allowed by the spectral fitting expanding atmosphere method. The intersection is the allowed range of rise-times and luminosities. The arrows denote the minimum possible rise-times allowed by observation for the indicated supernovaeKirshner et al. (1993); Branch et al. (1983a); Marvin & Perlmutter (1989); Richmond et al. (1995).

to compare to observations we must apply a bolometric correction to convert from M_{Bol} to the observationally accessible quantity, M_B. This quantity is a prediction of the models of Nugent et al. Nugent et al. (1995a), where the synthetic photometry is performed as described in Baron et al. Baron et al. (1995), however we have corrected an error that appeared in both those publications and will be described in detail elsewhere. We find $M_{\mathrm{Bol}} - M_B = 0.23$ mag. The bolometric correction is positive due to the fact that the flux from SNe Ia is highly non-Planckian; there is a large UV deficit due to line blanketing. Thus, the B-band is significantly brighter than it would be for a more nearly Planckian stellar spectral energy distribution of the same luminosity. Since the spectral energy distribution is the primary result of our modeling we are able to calculate the flux in any given filter band without additional error.

Applying this correction we find:

$$M_B = -19.47 \pm 0.45 \text{ mag.} \tag{3.10}$$

If we make the assumption that the normal SNe Ia that were used to derive this result (SNe 1981B, 1989B, 1992A and 1994D) are similar to the normal SNe Ia found on the ridge-line of the Hubble diagram for SNe Ia Vaughan et al. (1995, in press) we can apply Eq. 3.4 to obtain the Hubble constant:

$$H_0 = 57 \pm 11 \text{ km s}^{-1} \text{ Mpc}^{-1}. \tag{3.11}$$

3.7 Conclusions

Even normal SNe Ia are not entirely homogeneous, as demonstrated by observed correlations between absolute magnitude and photometric Hamuy et al. (1995); Sandage & Tammann (1995, in press); Riess, Press, & Kirshner (1995) or spectroscopic Fisher et al. (1995) properties of SNe Ia. Direct application of the methods discussed in this *Letter* to more distant SNe Ia would obviate the "standard candle" assumption and reduce the uncertainty in the value of H_0.

Our result for H_0 agrees with that of Höflich and collaboratorsHöflich et al. (1995); Höflich & Khokhlov (1996) and other SN Ia based methods Hamuy et al. (1995); Riess, Press, & Kirshner (1995) within the quoted errors, and indeed SNe Ia based methods are nearly converged.

Many astronomers find observational evidence that the value of the Hubble constant is high (≈ 80 km^{-1} s^{-1} Mpc^{-1}), which leads to the possibility of a cosmic "age problem" Hogan (1994); Pierce et al. (1994); Jacoby (1994); Freedman et al. (1994) and the need for a remedy such as a cosmological constant. On the other hand, SN Ia absolute magnitudes determined by means of Cepheid variables in the SN Ia parent galaxies Saha et al. (1994, 1995, in press) are consistent with our physically based estimate of the SN Ia absolute magnitude. Other physically based determinations of the value of the Hubble constant by techniques that are independent of local astronomical distance calibrations — time delay in a gravitationally lensed "double quasar" Dahle, Maddox, & Lilje (1994) and the Sunyaev–Zel'dovich effect in clusters of galaxies Jones et al. (1993); Birkinshaw & Hughes (1994) — also require low values of H_0.

We conclude that our present physical understanding of SNe Ia requires that the value of H_0 be low — and that the expansion age of the universe is not necessarily inconsistent with the ages of the oldest stars.